faculty of science

department of mathematics

Midterm II

MATH 232 D100 Spring 2012

Instructor: D. J. Katz

March 14, 2012, 11:30 a.m. – 12:20 p.m.

Name: ______________________________ (please print)

family name *given name*

SFU ID: ______________________________ @sfu.ca

student number *SFU-email*

Signature: ______________________________

Instructions:

1. Do not open this booklet until told to do so.
2. Write your name above in block letters. Write your SFU student number and email ID on the line provided for it.
3. Write your answer in the space provided below the question. If additional space is needed then use the back of the previous page. Your final answer should be simplified as far as is reasonable.
4. To receive full credit for a particular question you must provide a complete and well presented solution.
5. This exam has 5 questions on 5 pages (not including this cover page). Once the exam begins please check to make sure your exam is complete.
6. **No** calculators, books, papers, or electronic devices shall be within the reach of a student during the examination. Leave answers in "calculator ready" expressions: such as $3 + \ln 7$ or $e^{\sqrt{2}}$.
7. **During the examination, communicating with, or deliberately exposing written papers to the view of, or copying from, other examinees is forbidden.**

Question	Maximum	Score
1	8	
2	8	
3	11	
4	14	
5	9	
Total	50	

1. Let $\mathbf{u} = \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix}$, and $\mathbf{v} = \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix}$.

[2] (a) Compute $\mathbf{u} \times \mathbf{v}$.

[2] (b) Compute $(\mathbf{u} \times \mathbf{u}) \times \mathbf{v}$.

[2] (c) Compute $\mathbf{u} \times (\mathbf{v} \times \mathbf{u})$.

[2] (d) Find the area of the parallelogram determined by $\mathbf{u}$ and $\mathbf{v}$, i.e., where two of the sides are $\mathbf{u}$ and $\mathbf{v}$ with their initial points at the origin.

[2] **2.** (a) Compute $\frac{3+2i}{1+4i}$. Express your answer as $a+bi$, where a and b are real.

[3] (b) Compute $(\sqrt{3}+i)^7$. Express your answer as $a+bi$, where a and b are real.

[3] (c) Compute the determinant of $A = \begin{pmatrix} 1 & 1 & 3 & 0 \\ -2 & 2 & -5 & 1 \\ 0 & -3 & 2 & 1 \\ 0 & 1 & -1 & 4 \end{pmatrix}$.

3. We have a gas composed of molecules, each of which is in one of two states at any given time. The two states are the ground state and the excited state. We measure the fraction of the molecules in each of the two states once per minute. Each minute, $1/8$ of the molecules in the ground state change to the excited state, and $1/2$ of the molecules in the excited state change to the ground state. We begin the experiment at time $t = 0$ minutes with $2/3$ of the molecules in the ground state and $1/3$ of the molecules in the excited state.

[4] (a) Write a transition matrix A for this phenomenon.

[3] (b) Compute the fraction of molecules that are in the excited state one minute after the beginning of the experiment.

[4] (c) Find the long-term limit (as $t \to \infty$) of the fraction of molecules in the excited state.

4. Consider the linear operator T such that $T(1,0,0) = (1,1,0)$, $T(0,1,0) = (0,0,1)$, and $T(0,0,1) = (-1,1,0)$.

[2] (a) What is the domain of T? What is the codomain of T?

[3] (b) Find the matrix $[T]$, such that $T(\mathbf{x}) = [T]\mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^3$.

[2] (c) Find $T(2,1,-5)$.

[4] (d) Find a vector $\mathbf{x}$ such that $T(\mathbf{x}) = (3,1,2)$.

[3] (e) Is T an isometry (also known as an orthogonal operator)? Justify your answer.

5. Let $A = \begin{pmatrix} a & b & c \\ -3 & d & 3 \\ -9 & 0 & e \end{pmatrix}$ for some real numbers $a, b, c, d,$ and e. Suppose that A has eigenvalue 2 with algebraic multiplicity 1 and eigenvalue -1 with algebraic multiplicity 2. Also suppose that $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is an eigenvector of A with eigenvalue -1.

[3] (a) Compute $A^5 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

[6] (b) Find a, b, c, d, and e.